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Renormalisation-group invariance and universal soft supersymmetry-breaking

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We show that a particular "universal" form for the soft-breaking couplings in a softly broken $N = 1$ supersymmetric gauge theory is renormalisation-group invariant through two loops, provided we impose one simple condition on the dimensionless couplings. The universal form for the trilinear couplings and mass terms is identical to that found in popular derivations of the soft-breaking terms from strings or supergravity.

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If we take the standard model, generalise to two Higgs doublets, supersymmetrise, impose R-parity, and add all possible soft supersymmetry breaking terms then we have the supersymmetric standard model. The resulting theory has an alarming number of arbitrary parameters; far more than the standard model. It is customary to assume that the plethora of possible independent soft terms undergo a form of unification, at the same scale where the gauge couplings meet. At this scale it is supposed that the soft terms consist simply of a common scalar mass, a common gaugino mass, and ϕ^3 and ϕ^2 interactions proportional to the analogous terms in the superpotential; the constants of proportionality being denoted A and B respectively. This simplification can be motivated to some extent by appeal to $N = 1$ supergravity, and in particular to the idea that the supersymmetry breaking occurs in a hidden sector and is communicated to the observable sector via gravitational interactions (for a review, see [1]). It also arises in superstring phenomenology[2] [3].

In this note we attempt to motivate a simple form for the soft breakings in a different way. We explore the consequences of imposing that the soft breakings in the theory at the unification scale be form invariant under renormalisation. In other words we require that the theory be renormalisable, in the usual sense that counter-terms generated by shifting parameters and fields in the Lagrangian suffice to remove the divergences encountered in perturbation theory. In general, of course, imposing strict renormalisability requires us to write down all interactions permitted by the symmetries. We will find, however, that a particular universal form for the soft-breaking couplings (one which is compatible with the desired pattern of supersymmetry breaking described above) is renormalisation-group (RG) invariant at least through two loops provided we impose one simple condition on the dimensionless coupling sector of the theory. Theories with this property would have the attractive feature that the universal form of the soft breaking terms (which is presumably generated by supersymmetry breaking of the underlying supergravity or superstring theory at or near the Planck scale) would be exactly preserved down to the gauge unification scale. The Lagrangian $L_{\mathrm{SUSY}}(W)$ is defined by the superpotential

$$W = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j + L^i\Phi_i. \tag{1}$$

L_{SUSY} is the Lagrangian for the $N = 1$ supersymmetric gauge theory, containing the gauge multiplet $\{A_\mu, \lambda\}$ (λ being the gaugino) and a chiral superfield Φ_i with component fields $\{\phi_i, \psi_i\}$ transforming as a (in general reducible) representation R of the gauge group $\mathcal{G}$.

We assume that there are no gauge-singlet fields and that $\mathcal{G}$ is simple. (The generalisation to a semi-simple group is trivial.) The soft breaking is incorporated in $L_{\rm SB}$, given by

$$L_{\rm SB} = (m^2)^j_i \phi^i \phi_j + \left(\frac{1}{6}h^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{2}M\lambda\lambda + \text{h.c.}\right) \tag{2}$$

(Here and elsewhere, quantities with superscripts are complex conjugates of those with subscripts; thus $\phi^i \equiv (\phi_i)^*$.) Aside from the terms included in L_{SB} in Eq. (2), one might in general have $\phi^2\phi^*$-type couplings, $\psi\psi$ mass terms or $\lambda\psi$-mixing terms (as long as they satisfy a constraint that quadratic divergences are not produced). However, the soft-breaking terms we have included are those which would be engendered by an underlying supergravity theory and which are therefore considered most frequently in the literature.

The non-renormalisation theorem tells us that the superpotential W undergoes no infinite renormalisation so that we have, for instance

$$\beta_Y^{ijk} = Y^{ijp}\gamma^k{}_p + (k \leftrightarrow i) + (k \leftrightarrow j), \tag{3}$$

where γ is the anomalous dimension for Φ. The one-loop results for the gauge coupling β-function β_g and for γ are given by

$$16\pi^2\beta_g^{(1)} = g^3 Q, \quad \text{and} \quad 16\pi^2\gamma^{(1)i}{}_j = P^i{}_j, \tag{4}$$

where

$$16\pi^2 Q = T(R) - 3C(G), \quad \text{and} \tag{5a}$$

$$16\pi^2 P^i{}_j = \frac{1}{2}Y^{ikl}Y_{jkl} - 2g^2 C(R)^i{}_j. \tag{5b}$$

Here

$$T(R)\delta_{AB} = \text{Tr}(R_A R_B), \quad C(G)\delta_{AB} = f_{ACD}f_{BCD} \quad \text{and} \quad C(R)^i{}_j = (R_A R_A)^i{}_j. \tag{6}$$

The one-loop β-functions for the soft-breaking couplings are given by

$$16\pi^2\beta_h^{(1)ijk} = U^{ijk} + U^{kij} + U^{jki}, \tag{7a}$$

$$16\pi^2[\beta_{m^2}^{(1)}]^i{}_j = W^i{}_j + 2g^2(R_A)^i{}_j \text{tr}[R_A m^2], \tag{7b}$$

$$16\pi^2\beta_b^{(1)ij} = V^{ij} + V^{ji}, \tag{7c}$$

$$16\pi^2\beta_M^{(1)} = 2g^2 QM, \tag{7d}$$

where

$$
\begin{aligned}
U^{ijk} &= h^{ijl}P^k{}_l + Y^{ijl}X^k{}_l, && (8a)\\
V^{ij} &= b^{il}P^k{}_l + \frac{1}{2}Y^{ijl}Y_{lmn}b^{mn} + \mu^{il}X^j{}_l, && (8b)\\
W^j{}_i &= \frac{1}{2}Y_{ipq}Y^{pqn}(m^2)^j{}_n + \frac{1}{2}Y^{jpq}Y_{pqn}(m^2)^n{}_i + 2Y_{ipq}Y^{jpr}(m^2)^q{}_r \\
&\quad + h_{ipq}h^{jpq} - 8g^2MM^*C(R)^j{}_i, && (8c)
\end{aligned}
$$

with

$$
X^i{}_j = h^{ikl}Y_{jkl} + 4Mg^2C(R)^i{}_j. \qquad (9)
$$

Our assumption that the group $\mathcal{G}$ is semi-simple implies that the $\mathrm{tr}[R_Am^2]$ term in Eq. (7b) is zero, while the absence of gauge singlets means that (for instance in Eq. (8b)) we have

$$
Y_{ijk}b^{jk} = Y_{ijk}\mu^{jk} = 0. \qquad (10)
$$

We then claim that the conditions

$$
\begin{aligned}
h^{ijk} &= -MY^{ijk}, && (11a)\\
(m^2)^i{}_j &= \frac{1}{3}(1 - \frac{1}{16\pi^2}\frac{2}{3}g^2Q)MM^*\delta^i{}_j, && (11b)\\
b^{ij} &= -\frac{2}{3}M\mu^{ij} && (11c)
\end{aligned}
$$

are RG invariant through at least two loops, provided we impose the condition

$$
P^i{}_j = \frac{1}{3}g^2Q\delta^i{}_j. \qquad (12)
$$

(The idea of seeking relations amongst dimensionless couplings which are preserved by renormalisation has been explored in the coupling constant reduction programme of Zimmermann *et al.*[4].) We first demonstrate the RG invariance of the conditions Eq. (11). The invariance of Eq. (11a) requires

$$
\beta_h^{ijk} = -\beta_M Y^{ijk} - M\gamma^i{}_m Y^{mjk} - M\gamma^j{}_m Y^{imk} - M\gamma^k{}_m Y^{ijm}. \qquad (13)
$$

The strategy we adopt to verify equations such as Eq. (13) is to simplify the β-functions and anomalous dimensions as follows: firstly we use Eq. (12) to replace $P^i{}_j$ by Q. We also use Eqs. (11) to replace h^{ijk}, m^2 and b wherever they occur. Having done this, we find that any occurrences of $Y_{ikl}Y^{jkl}$, $C(R)$, $C(G)$ or $T(R)$ can be written in terms of P and Q

according to Eq. (5). We can now use Eq. (12) again if necessary to replace P by Q. For instance, we find, applying our strategy of imposing the condition Eq. (11a) in Eq. (9), and using Eqs. (5b), (12),

$$X^i{}_j = -MY^{ikl}Y_{jkl} + 4g^2MC(R)^i{}_j \\ = -\frac{2}{3}g^2QM\delta^i{}_j. \tag{14}$$

Henceforth we shall simply assume that this procedure is followed where possible. For instance, from Eqs. (8a), (14), we find

$$U^{ijk} = -g^2QMY^{ijk} \tag{15}$$

which, using Eqs. (7a, d), ensures that Eq. (13) is satisfied at one loop. The RG invariance of Eq. (11b) requires that

$$(\beta_{m^2})^i{}_j = \frac{1}{3}([1 - \frac{1}{16\pi^2}\frac{2}{3}g^2Q][\beta_M M^* + M(\beta_M)^*] - \frac{1}{16\pi^2}\frac{4}{3}g\beta_g QMM^*)\delta^i{}_j. \tag{16}$$

At one loop we readily find, from Eqs. (7b), (8c),

$$W^i{}_j = MM^*(4P^i{}_j - \frac{1}{16\pi^2}\frac{2}{3}g^2QY_{ikl}Y^{jkl}) \\ = g^2QMM^*(\frac{4}{3}\delta^i{}_j - \frac{1}{16\pi^2}\frac{2}{3}Y_{ikl}Y^{jkl}), \tag{17}$$

which, with Eqs. (7b, d) implies Eq. (16) at one loop. (The additional, two-loop term in Eq. (17) will be required later.) Finally, for the RG invariance of Eq. (11c) we need

$$\beta_b^{ij} = -\frac{2}{3}(\beta_M\mu^{ij} + M\gamma^i{}_k\mu^{kj} + M\gamma^j{}_k\mu^{ik}). \tag{18}$$

From Eqs. (8b), (14), we obtain

$$V^{ij} = -\frac{8}{9}g^2QM\mu^{ij}, \tag{19}$$

which, using Eqs. (7c, d), leads immediately to Eq. (18) at one loop. Finally, it behoves us to check that the condition Eq. (12) is itself RG invariant. This amounts to the condition

$$\frac{1}{2}\left(\gamma^i{}_mY^{mkl}Y_{jkl} + \gamma^m{}_jY^{ikl}Y_{mkl} + 4Y^{ikl}\gamma^m{}_lY_{jkm}\right) - 4g\beta_gC(R)^i{}_j = \frac{2}{3}g\beta_gQ\delta^i{}_j, \tag{20}$$

which is easily verified at one loop using Eqs. (4). The fact that the conditions Eq. (11), (12) are preserved by renormalisation at one loop seems to us remarkable enough; however,

they are actually preserved even at the two-loop level as well. The two-loop β-functions for the dimensionless couplings were calculated in Ref. [5]; they can be written in the form

$$(16\pi^2)^2\beta_g^{(2)} = 2g^5C(G)Q - 2g^3r^{-1}C(R)^i{}_jP^j{}_i \tag{21a}$$

$$(16\pi^2)^2\gamma^{(2)i}{}_j = [-Y_{jmn}Y^{mpi} - 2g^2C(R)^p{}_j\delta^i{}_n]P^n{}_p + 2g^4C(R)^i{}_jQ, \tag{21b}$$

where Q and $P^i{}_j$ are given by Eq. (5), and $r = \delta_{AA}$.

The calculation of the two-loop β-functions for the soft breaking couplings raises interesting issues concerning the use of dimensional reduction in non-supersymmetric theories [6].

The results are as follows[7]–[10]:

$$\begin{aligned}
(16\pi^2)^2\beta_h^{(2)ijk} = &-\Big[h^{ijl}Y_{lmn}Y^{mpk} + 2Y^{ijl}Y_{lmn}h^{mpk} - 4g^2MY^{ijp}C(R)^k{}_n\Big]P^n{}_p \\
&- 2g^2U^{ijl}C(R)^k{}_l + g^4(2h^{ijl} - 8MY^{ijl})C(R)^k{}_lQ - Y^{ijl}Y_{lmn}Y^{pmk}X^n{}_p \\
&+ (k \leftrightarrow i) + (k \leftrightarrow j),
\end{aligned} \tag{22a}$$

$$\begin{aligned}
(16\pi^2)^2[\beta_{m^2}^{(2)}]^j{}_i = &\Big(-\Big[(m^2)_i{}^lY_{lmn}Y^{mpj} + \frac{1}{2}Y_{ilm}Y^{jpm}(m^2)^l{}_n + \frac{1}{2}Y_{inm}Y^{jlm}(m^2)^p{}_l \\
&+ Y_{iln}Y^{jrp}(m^2)^l{}_r + h_{iln}h^{jlp} \\
&+ 4g^2MM^*C(R)^j{}_n\delta^p{}_i + 2g^2(R_A)^j{}_i(R_Am^2)^p{}_n\Big]P^n{}_p \\
&+ \big[2g^2M^*C(R)^p{}_i\delta^j{}_n - h_{iln}Y^{jlp}\big]X^n{}_p - \frac{1}{2}\big[Y_{iln}Y^{jlp} + 2g^2C(R)^p{}_i\delta^j{}_n\big]W^n{}_p \\
&+ 12g^4MM^*C(R)^j{}_iQ + 4g^4SC(R)^j{}_i\Big) + \text{h.c.},
\end{aligned} \tag{22b}$$

$$\begin{aligned}
(16\pi^2)^2\beta_b^{(2)ij} = &\Big[-b^{il}Y_{lmn}Y^{mpj} - 2\mu^{il}Y_{lmn}h^{mpj} - Y^{ijl}Y_{lmn}b^{mp} \\
&+ 4g^2MC(R)^i{}_k\mu^{kp}\delta^j{}_n\Big]P^n_p - \big[\mu^{il}Y_{lmn}Y^{mpj} + \frac{1}{2}Y^{ijl}Y_{lmn}\mu^{mp}\big]X^n{}_p \\
&- 2g^2C(R)^i{}_kV^{kj} + g^2C(R)^i{}_kY^{kjl}Y_{lmn}b^{mn} \\
&+ 2g^4(b^{ik} - 4M\mu^{ik})C(R)^j{}_kQ + (i \leftrightarrow j),
\end{aligned} \tag{22c}$$

$$(16\pi^2)^2\beta_M^{(2)} = g^2\Big(8g^2C(G)QM - 4r^{-1}C(R)^i{}_jP^j{}_iM + 2r^{-1}X^i{}_jC(R)^j{}_i\Big), \tag{22d}$$

where

$$S\delta_{AB} = (m^2)^i{}_j(R_AR_B)^i{}_j - MM^*C(G)\delta_{AB}. \tag{23}$$

The expressions given in Eq. (22) (and in particular Eq. (22b)) correspond to the use of a particular subtraction scheme whereby the mass of the ϵ-scalars decouples from the evolution of the other parameters. For a discussion, see refs. [9], [10].

At two loops we find, applying the usual procedure to Eqs. (21), (22d),

$$(16\pi^2)^2\gamma^{(2)i}{}_j = -\frac{2}{9}g^4Q^2\delta^i{}_j, \tag{24a}$$

$$(16\pi^2)^2\beta_g^{(2)} = -\frac{2}{3}g^5Q^2, \tag{24b}$$

$$(16\pi^2)^2\beta_M^{(2)} = -\frac{8}{3}g^4Q^2M. \tag{24c}$$

Now we can go on to check the RG invariance of Eqs. (11) to two-loop order. Using Eqs. (14), (15) in Eq. (22a), we find

$$(16\pi^2)^2\beta_h^{(2)ijk} = \frac{10}{3}g^4Q^2MY^{ijk}. \tag{25}$$

Inserting Eqs. (24a, c), and (25) into Eq. (13), we immediately verify the two-loop RG-invariance of Eq. (11a). Now using Eqs. (14), (17) in Eq. (22b), we obtain

$$(16\pi^2)^2(\beta_{m^2}^{(2)})^i{}_j = 4Qg^2MM^*(-Y_{ikl}Y^{jkl} + \frac{14}{3}g^2C(R)^i{}_j). \tag{26}$$

Hence, from Eqs. (7b), (17), (26), we obtain

$$(\beta_{m^2}^{(1)} + \beta_{m^2}^{(2)})^i{}_j = \frac{1}{16\pi^2}Qg^2MM^*(\frac{4}{3} - \frac{1}{16\pi^2}\frac{28}{9}g^2Q). \tag{27}$$

Using Eqs. (10), (27), (24c), (7d), (4) in Eq. (16), we see that Eq. (11b) is RG invariant throughtwo loops. Using Eqs. (14), (19), (12), (4) in Eq. (22c), we find

$$(16\pi^2)^2\beta_b^{(2)ij} = \frac{56}{27}g^4Q^2M\mu^{ij}. \tag{28}$$

On substituting Eqs. (28) and (24a, c) into Eq. (18), we see that Eq. (11c) is RG invariant at two loops. Finally, using Eqs. (24a, b), we verify Eq. (20) at two loops, ensuring the RG invariance of Eq. (12) at this level. Thus we have demonstrated the RG invariance of Eqs. (11) and (12) through two loops.

We turn now to the possibility of constructing realistic models satisfying our constraints. The main impact on low-energy physics, is that from Eq. (11) we have (in the usual notation) a universal scalar mass m_0 and universal A and B parameters related (to lowest order in g^2) to the gaugino mass M as follows:

$$m_0 = \frac{1}{\sqrt{3}}M, \tag{29a}$$

$$A = -M, \tag{29b}$$

$$B = -\frac{2}{3}M. \tag{29c}$$

Evidently it will be interesting to explore the region of the usual supersymmetric standard model parameter space consistent with Eq. (29); current experimental constraints will probably not rule out the scenario *per se*, but the various super-partner masses will be more tightly correlated than in the usual approach.

It follows from our results that if ${P^i}_j = Q = 0$, (guaranteeing that the dimensionless coupling β-functions are zero to two loops) then soft-breaking couplings related by Eq. (29) will also have vanishing β-functions, leading to the possibility of finite softly-broken supersymmetric theories. This has already been pointed out at the one-loop level in Ref. [11] and at the two-loop level in Ref. [9]. In Ref. [11] it was remarked that Eqs. $(29a, b)$ are consistent with the pattern of soft-breaking terms which emerges from supersymmetry breaking in the hidden sector of an underlying supergravity theory with a "minimal" Kähler potential. Even more interestingly, Eqs. $(29a, b)$ are identical to relations which arise in effective supergravity theories motivated by superstring theory, where supersymmetry breaking is assumed to occur purely via a vacuum expectation value for the dilaton[2][3]. More general scenarios involving vacuum expectation values for other moduli fields are also possible. To be more specific, we follow Ref. [3] in concentrating on the modulus T whose classical value gives the size of the manifold, and parametrising the ratio of the auxiliary fields F^S and F^T for the dilaton S and for T by an angle θ–so that θ characterises the extent to which supersymmetry-breaking is dominated by S or T. We also simplify still further by assuming a vanishing cosmological constant and by ignoring string loop corrections, and also the phases of F^S and F^T. In this more general case, the gaugino mass is related to the gravitino mass $m_{\frac{3}{2}}$ by $M = \sqrt{3} m_{\frac{3}{2}} \sin\theta$, and the soft-breaking parameters m_0 and A are still given by Eqs. $(29a, b)$, while B is either given by[3]

$$B = -\frac{M(1 + \sqrt{3}\sin\theta + \cos\theta)}{\sqrt{3}\sin\theta} \tag{30}$$

or[12]

$$B = \frac{2M}{\sqrt{3}\sin\theta}, \tag{31}$$

depending on whether the μ term is generated by an explicit μ-term in the supergravity superpotential, or by a special term in the Kähler potential. In the first case the value $\theta = \frac{4\pi}{3}$ reproduces our constraint Eq. $(29c)$; however, in the second case there is no value of θ which is consistent with this constraint.

In addition to Eq. (29), we also need to to impose Eq. (12) as a condition on the theory at the unification scale. It is not clear at present how such a constraint would

naturally emerge from string theory. The special case ${P^i}_j = Q = 0$, corresponding, as we have remarked, to two-loop finite theories, was tabulated in Ref. [13]. They found a fair number of possibilities, including a few of phenomenological interest: in particular a simple SU_5 model [14] [15].

We can anticipate, therefore, that it will be possible to construct unified models satisfying Eq. (12). The obvious try is a simple generalisation of the finite SU_5 model first analysed in ref. [14]. The superpotential is

$$W = \frac{1}{2}A_{ij\alpha}10_i 10_j H_\alpha + B_{ij\alpha}10_i\bar{5}_j\bar{H}_\alpha + C_{\alpha\beta}\bar{H}_\alpha 24 H_\beta + D24^3 \tag{32}$$

where $i, j : 1 \dots x$ and $\alpha, \beta : 1 \dots y$ so that we have x generations, y sets of Higgs multiplets $(H + \bar{H})$ and a single adjoint (24). It is straightforward to write down Eq. (12) for this model; tracing on all indices we obtain the relations:

$$|A|^2 + \frac{8}{5}|C|^2 = g^2 y\left(\frac{8}{5} + \frac{1}{9}Q\right), \tag{33a}$$

$$|B|^2 + \frac{6}{5}|C|^2 = g^2 y\left(\frac{6}{5} + \frac{1}{12}Q\right), \tag{33b}$$

$$|B|^2 = g^2 x\left(\frac{6}{5} + \frac{1}{12}Q\right), \tag{33c}$$

$$3|A|^2 + 2|B|^2 = g^2 x\left(\frac{36}{5} + \frac{1}{3}Q\right), \tag{33d}$$

$$\frac{189}{5}D^2 + C^2 = g^2\left(10 + \frac{1}{3}Q\right). \tag{33e}$$

where here $Q = 2x + y - 10$. It is easy to show, however, that Eqs. (33) do not have a solution unless $Q = 0$, which corresponds to the finite case. This outcome is not generic, however, and it is easy to modify the theory so as to produce candidate theories that do satisfy Eq. (12), for example by including one or more sets of $10 + \overline{10}$ multiplets. It remains to be seen, however, whether there exists a compelling unified theory with universal soft breaking terms. Meanwhile, if we conjecture that such a theory leads to the same low energy physics as the supersymmetric standard model, we can at least explore the consequences of Eq. (29) for the super-particle spectrum. We will report on these calculations elsewhere.

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